Back-Paper Exam B.Math III Year (Differential Geometry) 2016

Attempt all questions. This is a CLOSED BOOK and CLOSED NOTES exam. Results proved in class, or propositions (with or without proof) from the class notes maybe used after quoting the statements of those results. Results from exercises in the notes or Pressley's book, which haven't been solved in class must be proved in full if used.)

1. Let $c : I \to \mathbb{R}^3$ be a smooth curve of unit speed. Suppose that $\mathrm{Im}(c) \subset S^2$, where $S^2 \subset \mathbb{R}^3$ is the unit sphere centred at the origin. Show that the following are equivalent:

   (i): The torsion $\tau(t)$ of c is identically zero.

   (ii): c is the arc of a circle (not necessarily a great circle)

   (iii): The curvature $k(t)$ is a constant.

   (15mks)

2. Let $X \subset \mathbb{R}^3$ be a connected minimal surface (i.e. a surface of identically vanishing mean curvature). Show that X is a subset of a plane iff its scalar curvature $K(x)$ vanishes identically. (15mks)

3. Let $f : \mathbb{R} \to (0, \infty)$ be a smooth function and let X be the surface obtained by revolving the profile curve $x = f(z)$ about the z- axis. Show that the mean curvature of X is identically zero iff $f(z) = a \cosh\left(\frac{z-c}{a}\right)$ for some real numbers c, $a > 0$. That is, X is a catenoid. (15mks)